

**XCIII.** *A Letter from the Rev. Nevil Maskelyne, M. A. F. R. S. to the Rev. Thomas Birch, D. D. Secretary to the Royal Society: containing the Results of Observations of the Distance of the Moon from the Sun and fixed Stars, made in a Voyage from England to the Island of St. Helena, in order to determine the Longitude of the Ship, from Time to Time; together with the whole Process of Computation used on this Occasion.*

S I R,

St. Helena, Sept. 9, 1761.

Read June 24,  
and July 1,  
1762.

**D**uring the course of my voyage from England to this place, I made frequent observations of the distance of the Moon from the Sun and fixed stars, in order to determine our longitude: and, as from their agreement with each other, I humbly conceive it will be allowed, that the longitude may in general be ascertained by this method to sufficient exactness for nautical purposes, I flatter myself it may not be disagreeable to the Royal Society, if I communicate to them, through your hands, the results of my observations. I shall likewise deliver the whole process of computation, which I use in deducing the longitude from an observation, wherein I include several useful rules of my own investigation, which, I apprehend, render the calculation not only much shorter, but also much less intricate than it was before.

I am, S I R,

Your most obedient,

humble servant,

Nevil Maskelyne.

**M**Y Hadley's quadrant, which was made by Mr. Bird, the radius of which is 20 inches, appears, from all the trials I have made with it, to be very exact; this, indeed, the observations of the Moon alone sufficiently prove, since I have often taken the distance of the Moon from two stars, on different sides, on the same night, and found the same longitude to result from these separate observations, with as little difference as from two observations of the same star; whereas, if there was the least error in the division of the quadrant, or the least refraction in the glasses of it, the error arising from hence affecting the computation of the longitude from the two separate observations contrary ways, must be sensible in the result. I was secured from any errors in the construction of the quadrant, by the known skill of the artist; and the speculums, as well as dark glasses, were ground by Mr. Dollond, by a particular method of his own, by which he is certain of making the two surfaces of a glass truly parallel to each other.

The arch and the index were both of brass, and the frame of well seasoned mahogany; the Vernier's scale on the index carried the subdivisions to single minutes; and the eye might subdivide still nearer to the fraction of a minute. When the index was brought near the proper distance for observation, in order to give it a more steady motion than the hand alone could, a plate was screwed down to the arch, containing the head of a screw, by turning of which, the index was carried backwards or forwards at pleasure. This contrivance was extremely necessary for taking the distance of a star from the Moon's limb in

an exact manner; for the quadrant being often to be held in a very inclined plane, it was impossible, on account of the motion of the ship, to make the observation at once; therefore, by means of the screw, the index was moved by gentle degrees, and each time after altering the screw, the quadrant was turned by the hand round the visual ray, going to the star as an axis, so as to make the star seem to pass by the Moon's limb, and the index was gradually moved, till at last the star, in passing by, exactly swept the Moon's limb. Besides the screw moving the index, there was another addition no less necessary, which was a dark glass, much brighter than any of those commonly used, to take off the excessive brightness of the Moon, and that glare upon the horizon-glass, which always attends it. Without the use of this glass, it was not easy to observe accurately the distance of a star from the Moon's limb, except when the Moon's light was weakened by a thin cloud.

In order to render the contact of a star with the Moon's limb more discernible, I always used a small telescope, magnifying about four times, except when the motion of the ship was very troublesome, though I reckoned the observations were not quite so exact as those made with the telescope. The telescope, by means of two swivels adapted to it, had a motion parallel to itself, by which it was carried nearer to or further from the plane of the quadrant; by means of which, the Moon, which was seen always by reflexion, was rendered more or less bright, according as more or less of her light was received by the telescope, from the quicksilver part of the object speculum; or by raising the telescope still higher from  
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the plane of the quadrant, the Moon was seen entirely by reflexion from the unfoiled part of the speculum, which I generally found to be the most convenient: for my part, I should always recommend the using of a small telescope, magnifying about three times, as it would make the objects appear more distinct, and render the contact of the star with the Moon's limb more evident.

My telescope, which was about 6 inches long, consisted of two convex glasses, so that it inverted objects; which is of no sort of inconvenience to people, who have been used to inverting telescopes, the practice of which is soon acquired. But if any one would rather chuse to have a telescope that shews objects in their natural position, he may use one consisting of a convex object-glass and a concave eye-glass; which kind of telescope answers better in short lengths than the other.

Before I deliver the process of the computation of the longitude, I shall first say something with respect to the observations themselves, and mention some cautions concerning them.

A most particular attention must be paid to the exact adjustment of the quadrant, as a thing of equal consequence with the observation of the distance of the star from the Moon. This is so much the more necessary, as the adjustment is subject to alter sensibly, even from one day to another. The best object of all for this purpose is the horizon of the sea, when clear; and I found it most convenient, in this case, not to use the telescope, but applied a concave glass to my eye, which was fitted for giving

me the most distinct view of distant objects, which also I always used in all observations where the telescope was not applied. The observer should always be careful to examine the adjustment of his quadrant in the day-time, when the horizon is to be seen clearly, and particularly in the afternoon, when he expects to make an observation the ensuing night. Should he have failed of making this examination the afternoon before an observation, he may examine what the error was, if any, the next morning; or he may make this examination by means of the Moon itself, in which case, it will be best to use the telescope; though the horizon of the sea is, in general, by far the best object for this purpose. As I found my quadrant would seldom continue exactly adjusted for twenty-four hours together, instead of fresh adjusting it every day, I chose rather to examine what the error of the adjustment itself was. This is done by turning the index of the quadrant, till the horizon of the sea, or the Moon, or any other proper object, appears as one, and then the number of minutes, by which  $\circ$  on the index differs from  $\circ$  on the arch, is the error of the adjustment. If  $\circ$  on the index stands advanced upon the quadrant, before or to the left hand of  $\circ$  on the arch, that number of minutes is to be subtracted from all observations; but if it stands off the arch behind, or to the right hand of  $\circ$  on the arch, it must be added to the observations. By examining the error of the adjustment in this manner, by at least three trials, and taking a medium of the results, one can scarce err above half a minute in determining the exact error of the quadrant; whereas one may be mistaken a minute, or more, by a single trial.

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The observer would do well to write down in a book the error of adjustment thus found, and which way it is to be applied, whether it is additive or subtractive.

As it is necessary to know the exact time of the day, when the distance of the star from the Moon is taken, which can only be found at sea, by an altitude of the Sun, or a star; and as the altitude of a proper star cannot always be taken with sufficient exactness in the night for this purpose, I would recommend it to the observer, when he expects to observe the Moon at night, to take an altitude of the Sun in the afternoon, two or three hours from noon, the more the better, provided the Sun be not too near the horizon; noting the exact time shewn by his watch, at the instant of observation; which will shew him, by computation, how much his watch is too fast or slow for the meridian of the place; whence the time shewn by the watch being also noted, when the distance of the star from the Moon was taken, he will know what the exact time of the day then was. Or if an altitude of the Sun was not taken the evening before, an altitude might be taken the next morning, after the observation of the Moon, provided the interval of time between the observation of the Moon, and the Sun's altitude did not exceed twelve hours; for a good common watch would hardly vary above a minute during that space of time, which only makes an error of a quarter of a degree of longitude.

The time being determined by an altitude of the Sun or a star, and the distance of a proper star from the Moon's limb, or the distance of the Sun and Moon's nearest limbs in the first and last quarter, being  
carefully

carefully observed, the longitude may be found from thence, without any other observations: and this is the method proposed by the late Dr. Halley, which certainly deserves to be highly esteemed for its great simplicity, and the small number of observations which it requires. Nevertheless, I must own myself of opinion with the learned Abbé De la Caille, that it will be more convenient at sea to require the aid of more observations, which is the method I constantly practised myself, during my voyage, having always two observers, who were ready, one to take the altitude of the star, and the other of the Moon's upper or lower limb, at the instant I spoke when I had made the observation of the distance of the star from the Moon.

I can therefore answer, from my own experience, both that the method is practicable at sea, and also, that so far from being less simple, it is more so than the other method; for the additional observations that it requires are very easily made, and even the error of a degree in the altitudes would seldom be of more consequence than an error of a minute, in taking the distance of the star from the Moon; so that an error of 10' or 15' in the altitudes would be of no great prejudice: but with respect to the facility of the calculations, there is no comparison between the methods, the latter being much less intricate, and much more concise. The Abbé De la Caille requires the altitude of that part of the Moon's limb from which the distance of the star is taken; but as at sea we can only take the altitude of the Moon's upper or lower limb, an allowance might be made near enough, by estimation of the eye, for the difference of altitude  
between

between the Moon's upper or lower limb, and that part of the limb from which the distance of the star is taken, I generally added the semidiameter of the Moon to, or subtracted it from, the observed altitude of the lower or upper limb, in order to have the apparent altitude of the center, and I found the apparent distance of the star from the Moon's center, by adding or subtracting the Moon's horizontal semidiameter, augmented according to her height, to or from the observed distance of the star from the Moon's nearest or remotest limb.

This method will be exact enough, if the altitude of the Moon or star be not less than  $5^{\circ}$ . Having thus got three sides of the spherical triangle formed by the Moon, the star, and the zenith; namely, the apparent zenith distance of the Moon, the apparent zenith distance of the star, and the distance of the star from the Moon, I find the effect of refraction and parallax, in altering the apparent distance of the star from the Moon, by the two following rules:

#### R U L E I.

To find the effect of refraction in contracting the apparent distance of two stars, or of the Moon and a star.

Add together the logarithm-tangents of half the sum, and half the difference of the two zenith distances, the sum abating 10 from the index is the tangent of arc the first. To the logarithm-tangent just found, add the logarithm-cotangent of half the distance of the two stars, the sum abating 10 from the index is the tangent of arc the second. Then  
add



add together into one sum the logarithm-tangent of double the first arc, the co-secant of double the second arc, and the constant logarithm 2.0569; the sum abating 20 from the index is the logarithm of the number of seconds required; by which the distance of the stars, or of the Moon and stars, is contracted by refraction: which therefore, added to the observed distance, gives the true distance, cleared from refraction.

N. B. This rule may be made universal, so as to serve with equal exactness almost down to the horizon, if the apparent zenith distances be diminished by three times the refraction belonging to them, found from any common table of refraction, and the computation be made with the zenith distances thus corrected. But if the altitudes of the Moon and star be not less than  $10^{\circ}$ , this correction will not be necessary. It will not be proper to make the observations, if the altitudes of the star and Moon are either of them less than  $4^{\circ}$  or  $5^{\circ}$ ; on account of the variableness of refraction near the horizon.

#### R U L E II.

To find how much the distance of the Moon and a star is increased or diminished, on account of the Moon's parallax.

Add together into one sum the logarithm-tangents of half the sum, and half the difference of the zenith distances, and the cotangent of half the distance of the Moon and star, all corrected for refraction; the sum, abating 20 from the index, is the tangent of arc the

the third, for which arc the second, found by the first rule, may be taken, without any sensible error.

Then, if the zenith distance of the Moon is greater than that of the star, take the sum of this arc, and half the distance of the Moon and star; but if the zenith distance of the Moon is less than that of the star, take the difference of the said arcs; the tangent of the sum or difference, which may be called the parallaxic arch, added to the cosine of the Moon's zenith distance, and the logarithm of the Moon's horizontal parallax in minutes, abating 20 from the index, is the logarithm of the number of minutes required, by which the apparent distance of the Moon from the star is always augmented by parallax, unless the zenith distance of the star be greater than that of the Moon, and, at the same time, arc the third be greater than half the distance of the Moon and star; in which case, the apparent distance of the Moon and star is diminished by the parallax:

Therefore, the number of minutes found by this rule is always to be subtracted from the observed distance of the Moon and star, first corrected for refraction, in order to find the true distance, cleared from the effect of parallax likewise; except in the case specified, when the zenith distance of the star is greater than that of the Moon, and arc the third is at the same time greater than half the distance of the Moon and star, when the correction is to be added. In computing these corrections, four places of figures, besides the index, will be sufficient.

It remains to be found, by calculation, at what hour under a known meridian, the distance of the Moon from the star will be the same as results from

the observation, cleared of refraction and parallax. For this purpose, it is necessary to compute the Moon's longitude and latitude, and horary motion both in longitude and latitude, from the most exact tables for the time under the known meridian, which is judged to correspond nearly to the given time of observation under the unknown meridian. The mean motions of the Sun and Moon I took from very exact tables, which I received as a present from the ingenious Mr. Gael Morris, composed by himself, from the comparison of a great number of Dr. Bradley's observations; to which I applied the lunar equations, as they stand in the learned Mr. Mayer's printed tables. After finding the mean longitude of the star at the present time, I always allowed for its aberration in longitude, which will sometimes amount to  $20''$ , without considering the aberration in latitude, which can be of no consequence in a zodiacal star, such as those are which are always to be used in these observations. The distance of the star from the Moon I computed from their longitudes and latitude, by the two following rules:

#### R U L E I.

Add together the logarithmic cosine of the difference of the computed longitudes of the Moon and star, and logarithmic cosine of the difference of their latitudes, if they are of the same denomination; or sum, if they are of different denominations; the sum, abating 10 from the index, is the cosine of the approximate distance.

N. B. This gives the absolute distance of the Moon from the Sun, without any further calculation.

But in case of a star, it is necessary to apply another rule also. Seven places of logarithms, besides the index, must be used, in computing from this rule, and the calculation must be carried to seconds.

#### R U L E II.

To the constant logarithm 3.5363, add the sines of the Moon's and star's latitudes, the versed-sine of the difference of longitude, and the co-secant of the approximate distance just found; the sum, abating 40 from the index, is the logarithm of a number of minutes, to be subtracted from the approximate distance, to find the true distance, if the latitudes of the Moon and star are of the same denomination; but to be added, if they are of contrary denominations. The second of these two rules, though only an approximation, is so exact, that if the latitude of the Moon was  $5^\circ$ , and that of the star  $15^\circ$ , the error resulting would be only  $10''$  in the distance. Four places of figures will be sufficient in computing from this rule.

If the distance of the Moon from the star thus computed, at the assumed time under a known meridian, suppose Greenwich, agrees with the distance observed, corrected for refraction and parallax, the time at Greenwich was assumed right, and the difference between this time and the time of the observation under the unknown meridian, is the difference of longitude in time between the said meridian and Greenwich; which is turned into degrees and minutes of the equator, by allowing  $15^\circ$  for every hour, and  $1^\circ$  for every four minutes of time.

But if the distance computed differs from the distance inferred from the observation, it must be found by proportion from the Moon's horary motion to or from the star, how long time she will take to run over that difference; whence the time will be found at Greenwich, when the true distance of the Moon from the star was the same with that resulting from the observation; which, compared with the time of the observation by the meridian of the ship, gives the difference of longitude from Greenwich, as before. If the distance of the Moon from the star computed agrees with that resulting from observation within 10' or 12', and the distance of the Moon from the star be not less than 20° or 30°, the horary motion of the Moon in the ecliptic may be taken for the horary motion of the Moon to or from the star; but otherwise, the Moon's longitude and latitude must be found at an hour's interval after the time assumed at Greenwich, by adding the horary motions to the longitude and latitude computed; and by the application of the rules, the distance of the star from the Moon must be found again at the end of that hour; which gives the horary motion to or from the star, as required.

It is to be observed, that the longitude thus found, is that of the ship, at the instant when the altitude of the Sun or star was taken, by which the watch was regulated, and not at the time of the observation of the distance of the star from the Moon; for the watch being supposed not to vary considerably during that interval of time, must continue, to indicate the time according to the meridian, by which it was corrected; and the observation of the distance of the Moon from the star shewing the time at Greenwich, the difference must

must shew the difference of longitude between that meridian and Greenwich.

Perhaps the following method of deducing the longitude from the observations may be least liable to mistake :

Find what the longitude by account was, at the instant of taking the Sun's or star's altitude, for the regulation of the watch ; which being turned into time, at the rate of one hour for every  $15^{\circ}$ , and four minutes for every degree, add to the correct time from noon, when the distance of the star from the Moon was taken, if the ship is to the west of Greenwich, or subtract from it, if it be to the east : this gives the apparent time at Greenwich by account ; and the mean time is found, by applying the equation of time ; to which time, compute the Moon's longitude and latitude from the tables, and the distance of the star from the Moon, by the rules, and find, by proportion, as before, what time the Moon will take to run over the difference between the distance computed, and that resulting from the observation ; this turned into degrees and minutes of the equator, will shew the error of the ship's account ; and the following rules will show, whether the ship is to the east or west of its account :

If the distance of the Moon observed east of a star (or the Sun in the first quarter) is greater than that computed, the ship is west of the longitude by account ; but if the distance observed is less than that computed, it is east of account.

If the distance of the Moon observed west of a star (or the Sun in the last quarter) is greater than computed, the ship is east of account ; but if the distance  
observed

observed is less than computed, it is west of account.

The horary motion of the Moon in the ecliptic may be thus made out very expeditiously from Mayer's equations, by the help of the principal arguments used in the computation of the Moon's place. Call A, B, C, and D, the differences of the equations of the center, evection, and variation, and reduction to the ecliptic, for  $1^\circ$  addition to their arguments; where it must be noted, that they must have the same sign as the equation, if it is increasing; but a contrary sign, if it is decreasing. Compute the value of  $32' 56'' + A \times \frac{1}{2} \times \frac{99}{100} + B \times \frac{1}{2} \times \frac{16}{17}$ , which put  $= H$ ; and the true horary motion of the Moon in her orbit  $= H + C \times \frac{H - 2' 28''}{60'}$ , which put  $= K'$ ; and the horary motion of the Moon in the ecliptic is  $K + \frac{D \times K'}{60'}$ .

The horary motion of the Moon in latitude, calling the difference, answering to  $1^\circ$  increase of the argument of latitude E, is  $\frac{E \times K'}{60'}$ .

The most difficult part in the above computations, and in which a person is most liable to make mistakes, is the computation of the Moon's place; but if this be done at land for every twelve hours at least, and the distance of a proper star, or of two stars, one to the east, and the other to the west, from the Moon's enlightened limb, be computed for every six hours at least, according to Mons. De la Caille's proposal, the rest of the computation, which will remain to be done at sea, will be very plain and concise.

Here

Here follows the series of my determinations of the longitude, during my voyage, delivered in an extract of my sea journal. The first column contains the day of the month; the second, the latitude; the third, the longitude, which I deduced from my observations of the Moon, reduced to the nearest noon; the fourth shews the longitude per account, kept in the usual manner; the fifth gives the difference between the third and fourth columns, and expresses how much the longitude deduced from the observation of the Moon is west of the longitude per account; the last column shews, whether the distance of the Sun from the Moon, or distance of what star from the Moon, was observed.

1761.  
♂ Jan. 20.

At noon, took our departure from the Lizard, which bore full north, distance 21 miles, allowing its longitude from London to be  $5^{\circ} 14'$  west, and latitude  $49^{\circ} 57'$  north.

	Latitude:	Longitude W. by obser- vation of Moon.	Longitude per reckon- ing.	Longitude by Moon W. of account.	The Sun or stars whose distance from the Moon's enlightened limb was taken.
	o /	o /	o /	o /	
♂ Feb. 10.	16 49 N.	30 22 W.	27 33 W.	2 49 W.	Sun's E. limb from Moon's W. limb.
11.	14 3 N.	29 22 W.	26 47	2 35 W.	Sun's E. limb from Moon's W. limb.
15.	5 10 N.	23 39 W.	22 44	0 55 W.	Cor Leonis from Moon's W. limb.
19.	1 42 N.	23 35	22 44	0 51 W.	Pollux from Moon's E. limb.
23.	9 6 S.	29 44	26 2	3 42 W.	Sun's W. limb from Moon's E. limb.
March 9.	24 9 S.	30 7	25 55	4 12	Aldebaran from Moon's W. limb.
10.	25 51 S.	29 32	24 32	5 0	Aldebaran from Moon's W. limb.
13.	29 49	27 55	22 19	5 36	Sun's E. limb from Moon's W. limb.
		27 44		5 25	Cor Leonis from Moon's W. limb.
15.	30 8	26 52	22 8	4 44	Cor Leonis from Moon's W. limb.
17.	30 39	24 50	18 58	5 52	Pollux from Moon's W. limb.
		25 5		6 7	Spica Virginis from Moon's W. limb.
18.	31 1	24 0	18 47	5 23	Pollux from Moon's W. limb.
		24 33		5 46	Spica Virginis from Moon's W. limb.
19.	31 37	21 41	17 7	4 34	Pollux from Moon's W. limb.
20.	32 4	20 19	15 27	4 52	Pollux from Moon's E. limb.
		22 4		6 37	Antares from Moon's E. limb.
25.	31 23	12 4	5 57	6 7	Spica Virginis from Moon's E. limb.
26.	30 51	9 55	4 17	5 38	Spica Virginis from Moon's E. limb.
29.	30 32 S.	6 49 W.	1 13 W.	5 36	Sun's W. limb from Moon's E. limb.
April 6.	7 $\frac{1}{2}$ A. M. came to an anchor in the harbour before James's fort at St. Helena; making the latitude per account $15^{\circ} 55'$ S. longitude per common reckoning $1^{\circ} 28'$ E. longitude corrected by observations of the Moon $4^{\circ} 16'$ W.				

By



By the comparison of the longitude determined by the Moon, with the longitude by the common reckoning, we seem to have been set by a current to the eastward about 20 miles per day, between February 10th and 15th, while we were passing from  $17^{\circ}$  to  $5^{\circ}$  north latitude, at the distance of about  $11^{\circ}$  westward of the coast of Guinea; and I am told that ships passing near the coast of Guinea always meet currents, which set them in upon the land, which are so much the stronger the nearer they approach the coast; as on the contrary, if they approach the opposite coast of the Brasils, they will be set by a current to the westward, which appears to have been our case; for between February 19th and 28th, during which time we passed by the most eastern part of the coast of Brasils, leaving cape St. Augustin only  $6^{\circ}$  to the west of us, we appear to have been set by a current at the rate of 20 miles per day to the westward, and from this time to our arrival at St. Helena, we seem to have been continually set to the westward, though slower than before; which must have been owing to our approaching so much nearer, and continuing so much longer near the eastern coast of South America, than the western coast of Africa.

Though I had no observation of the Moon within less than eight days of my arrival at St. Helena, I make the longitude of the island, by my account, to be only  $1\frac{1}{2}^{\circ}$  east of its true situation, which is  $5\frac{3}{4}^{\circ}$  west of London; whereas the account kept in the common manner made the island  $1\frac{1}{2}^{\circ}$  east of London, or  $7\frac{1}{4}^{\circ}$  east of its true situation, and most of the accounts on board the ship made it  $10^{\circ}$  east of the true longitude. Having got twelve observations in the  
compass

compass of eleven days, between March 9th and 20th, I had the curiosity to compare them together. Setting aside any errors in the common method of keeping a ship's reckoning, and supposing us not to be affected by currents during this time, the same difference ought to have been found between all the longitudes by account, and the longitudes deduced from the Moon, or the same error of the common account ought to have resulted from all the observations. The mean error of account from all the twelve observations is  $5^{\circ} 20'$ , by which we were really more to the west than our account made us. And comparing each particular error of account with this quantity, the difference between them, in any of the twelve observations, scarce exceeds a degree; whence we may suppose, that the longitude was deduced truly from every one of these twelve observations, within the compass of less than  $1\frac{1}{4}^{\circ}$ .

I have set them down in the following table, the error of the common account, and the difference between  $5^{\circ} 2'$ , the mean quantity of it, and each particular error of account, which, except in the first and last observations, does not exceed  $\frac{3}{4}^{\circ}$ . The last observation, which differs most from the medium, was taken in some haste, on account of the position of the sails of the ship, which did not allow a mean uninterrupted view of the star; nevertheless, as I was tolerably satisfied with the observation at the time, and as it does not materially differ from the others, I did not think proper to reject it.

	Error of account.	Difference between 5° 20', and each particular error of account.
1761.	° /	° /
March 9.	4 12	— 1 8
10.	5 0	— 0 20
13.	5 36	+ 0 16
	5 25	+ 0 5
15.	4 44	— 0 36
17.	5 52	+ 0 32
	6 7	+ 0 47
18.	5 13	— 0 7
	5 46	+ 0 26
19.	4 34	— 0 46
20.	4 52	— 0 28
	6 37	+ 1 17
Mean error of account	5 20	

After finding so great an agreement in the result of all my different observations, whether made on the same or different stars, or on the same or different nights, I must own I find myself at a loss to account for the great difference found by the Abbé De la Caille, in the result of several observations taken by himself, and a friend of his, at land, which ought to agree still nearer with one another than those made at sea. I cannot conceive that such able observers could be liable to an error of 5' in measuring the distance of a star from the Moon's limb, if their instruments were not faulty. The most likely and the most common cause of error lies in the speculums and dark glasses;

glasses; for if these are not ground truly parallel, which I am afraid they very often are not, by the common methods, they may easily produce a refraction of some minutes.

As a proof how near different observations made in compass of an hour or two will agree in giving the same longitude, February 11th, by ten different observations of the distance of the Moon from the Sun, I made the longitude, reduced to noon as usual,  $28^{\circ} 57'$ ,  $29^{\circ} 50'$ ,  $29^{\circ} 16'$ ,  $29^{\circ} 22'$ ,  $29^{\circ} 53'$ ,  $28^{\circ} 59'$ ,  $29^{\circ} 30'$ ,  $29^{\circ} 48'$ ,  $29^{\circ} 30'$ ,  $29^{\circ} 30'$ : none of which differ above half a degree from  $29^{\circ} 22'$ , which is the medium of them all. March 18th, by four different observations of the distance of Pollux from the Moon, I found the longitude  $23^{\circ} 52'$  twice, and  $24^{\circ} 8'$  twice. I never found that a single observation would give the longitude above a degree different from the medium resulting from three or four observations, and seldom above half a degree; which argues, that the error of any single observation never exceeded two minutes, and seldom one minute.

From the whole, I congratulate the curious astronomer and ingenious mariner, that the method of finding the longitude, proposed by Sir Isaac Newton, is by the improvement of the theory, of which he laid the foundation, and, by the great perfection to which our artists have carried the construction of instruments, rendered practicable in our times, at sea as well as at land, to a degree of exactness sufficient to make it of great and valuable utility to the extensive navigation and commerce of our native country.